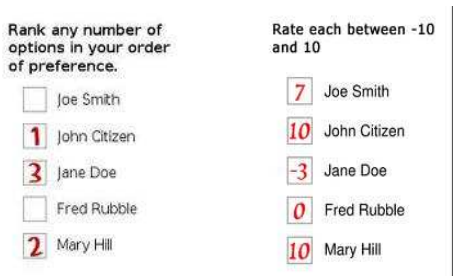
11.2 Defects in Voting Methods



Recall the different Voting Methods:

1. Plurality - one vote to one candidate, the others get nothing

The remaining three use a preference ballot, where all candidates are ranked.

2. Borda Count – points assigned to candidates for being 1st, 2nd, etc and then the candidate with the highest total points wins.

3. Plurality-with-Elimination – remove the candidate with the least 1st place votes. Repeat until only one remains, the winner.

4. Pairwise Comparison – For every pair of candidates, give 1 point to the one who would win in a two candidate election, ½ point if they tie. The one with the highest total points is the winner.

We will analyze how these well intended methods can go wrong.

Majority = more than half

DEFINITION The Majority Criterion
If a majority of the voters rank a candidate as their first choice, then that candidate should win the election.

This works for the plurality method.

For Elimination method, if B gets more than half of the 1st place votes, B will never be eliminated and thus will win.

The Elimination method satisfies this criterion.

Consider the pairwise comparison method.

If B is the 1st choice more for more than half the ballots, then B will always win 1 point for each pair tested (with B in it).

So B will get the highest points and win.

Pairwise comparison satisfies the majority criterion.

Testing Borda Count method.

Voter 1: A B C D
Voter 2: B D C A
Voter 3: A B D C

Testing Borda Count method.

Voter 1: A B C D
Voter 2: B D C A
Voter 3: A B D C

A gets 4 + 1 + 4 = 9 points
B gets 3 + 4 + 3 = 10 points
C gets 2 + 2 + 1 = 5 points
D gets 1 + 3 + 2 = 6 points

B wins, but A had the most 1st place votes.

Borda Count violates the Majority Condition!

DEFINITION Condorcet's Criterion
If candidate X can defeat each of the other candidates in a head-to-head vote, then X is the winner of the election.

This is an idea behind the pairwise comparison method. So it works.

Testing Pluriality method.

Voter 1: C A B
Voter 2: B A C
Voter 3: A B C
Voter 4: B A C
Voter 5: B A C
Voter 6: C A B
Voter 7: A B C

Testing Pluriality method.

Voter 1: C A B
Voter 2: B A C
Voter 3: A B C
Voter 4: B A C
Voter 5: B A C
Voter 6: C A B
Voter 7: A B C

B wins plurality (most 1st place votes)
A beats B as a pair, and A beats C as a pair.

Plurality fails Condorcet's Criterion.

DEFINITION **Independence-of-Irrelevant-Alternatives Criterion**
If candidate X wins an election, some nonwinners are removed from the ballot, and a recount is done, then X still wins the election.

This is similar to the Elimination method.

Testing Pluriality method.

Voter 1: A B C
Voter 2: C B A
Voter 3: A C B
Voter 4: B A C
Voter 5: B C A
Voter 6: C B A
Voter 7: A B C

Testing Pluriality method.

Voter 1: A B C
Voter 2: C B A
Voter 3: A C B
Voter 4: B A C
Voter 5: B C A
Voter 6: C B A
Voter 7: A B C

Remove C and now B has more 1st place votes than A.

Plurality fails this condition.

• Example: The following table summarizes the preference ballots cast for candidates A, B, C, and D: Determine the winner of this election using pairwise comparison. Do the results of the election change if any of the losing candidates are removed?

Preference	Number of Ballots			
	8	4	5	1
1st	A	D	C	D
2nd	B	A	B	A
3rd	C	C	D	B
4th	D	B	A	C

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• Solution: We note that A is the winner in a head-to-head vote.

	Vote Results	Points Earned
A vs. B	A wins 13 to 5.	A gets 1 point.
A vs. C	A wins 13 to 5.	A gets 1 point.
A vs. D	D wins 10 to 8.	D gets 1 point.
B vs. C	Tie—each has 9.	B and C get $\frac{1}{2}$ point.
B vs. D	B wins 13 to 5.	B gets 1 point.
C vs. D	C wins 13 to 5.	C gets 1 point.

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Removing B and C, we see that D defeats A by 10 votes to 8. This method does not satisfy the independence-of-irrelevant-alternatives criterion.

Preference	Number of Ballots			
	8	4	5	1
1st	A	D	D	D
2nd	D	A	A	A

DEFINITION **The Monotonicity Criterion**
If X wins an election and in a reelection all voters who change their votes only change their votes to favor X, then X also wins the reelection.

Plurality satisfies this criterion since if a candidate who wins gets more votes, that candidate still wins.

The Monotonicity Criterion

- Example: An election for president of a club has (C)hang, (K)wami, and (W)oytek as candidates. Plurality-with-elimination is being used to determine the winner. Three supporters of W, who had preferred C, decide to support her in the election. W tells the new supporters to vote for C instead. If the three voters indicated in the highlighted column in the table (next slide) change their votes to W first, C second, and K third, why should this cause W concern?

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The Monotonicity Criterion

- Solution:

K has the fewest 1st place votes and is eliminated. W wins the election.

Preference	Number of Ballots			
	12	9	3	8
1st	W	K	C	C
2nd	C	W	W	K
3rd	K	C	K	W

Preference	Number of Ballots			
	12	9	3	8
1st	W		C	C
2nd	C	W	W	
3rd		C		W

Woytek wins 21 to 11.

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The Monotonicity Criterion

Now consider the situation if the three voters had changed their votes. In this case, C has the least votes and is eliminated.

Preference	Number of Ballots			
	12	9	3	8
1st	W	K	W	C
2nd	C	W	C	K
3rd	K	C	K	W

Chang is eliminated.

These three voters have changed their votes.

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The Monotonicity Criterion

With C eliminated, K now wins the election.

Preference	Number of Ballots			
	12	9	3	8
1st	W	K	W	
2nd		W		K
3rd	K		K	W

Kwami wins 17 to 15.

The Monotonicity Criterion

Flaws in voting methods.

	Plurality	Borda Count	Plurality with Elimination	Pairwise Comparison
Majority	Yes	No	Yes	Yes
Condorcet's	No	No	No	Yes
Independence-of-irrelevant-alternatives	No	No	No	No
Monotonicity	Yes	Yes	No	Yes

PROBLEM SOLVING

Arrow's Impossibility Theorem

In any election involving more than two candidates, there is no voting method that will satisfy all of the four fairness criteria.